Application of Derivative

Maxima and Minima, Demand, Cost, Revenue, Profit function:

Cost function = $C(x)$

1. Cost Function:-

(a) Average Cost (AC) = $\frac{C(x)}{x}$ = $\frac{\text{Total Cost}}{\text{output}}$

(b) Average variable cost (AVC) = $\frac{V(x)}{x}$ = $\frac{\text{variable Cost}}{\text{output}}$

So. $C(x) = F + V(x)$.

2. Revenue function:

The Revenue function denoted by $R(x)$, is the total amount of money generated from the sale of x unit of an item by a firm.

If x units be sold at ₹p per unit, then

$R(x) = px$ where $p > 0$, $x > 0$

Average Revenue (AR) = $\frac{R(x)}{x}$ = Price per unit

3. Profit Function:- Profit function is denoted by $P(x)$ from the sale of x units after all cost have been deducted.

$P(x) = R(x) - C(x)$

For break even point $P(x) = 0$ and

$R(x) = C(x)$

4. Marginal Cost (MC) = $\frac{dc}{dx}$ where $c = C(x)$

5. Marginal Revenue (MR) = $\frac{dR}{dx}$

6. Curvature:-

The Curvature (K) of the curve $y = f(x)$ at any point $P(x,y)$

is given by $K = \frac{|y_2|}{(1+y_1^2)^{3/2}}$, where $y_1 = \frac{dy}{dx}$ and $y_2 = \frac{d^2y}{dx^2}$

or $K = \frac{x_2}{(1+x_1^2)^{3/2}}$ where $x_1 = \frac{dx}{dy}$ and $x_2 = \frac{d^2x}{dy^2}$

We shall consider numerical value of curvature,

Radius of Curvature = $\rho = \frac{1}{K}$

1. A firm produces x tonnes of output at a total Cost $C(x) =$ Rs. $(\frac{1}{10}x^3 - 6x^2 + 70x + 11)$. Find the Average cost, Average variable cost, and Average fixed cost in terms of x. Find the value of each of these at the level of output of 10 tonnes.

Soln.:- Here Total cost $C(x) = Rs.(\frac{1}{10}x^3 - 6x^2 + 70x + 11)$

So Average Cost (AC) $= \frac{C(x)}{x} = Rs\ (\frac{1}{10}x^2 - 6x + 70 + \frac{11}{x})$

variable Cost (VC) $= Rs.(\frac{1}{10}x^3 - 6x^2 + 70x)$

Fixed Cost (FC) $= Rs. 11$ [exclusive x]

Average variable cost (AVC) $= \frac{VC}{x} = \frac{\frac{1}{10}x^3 - 6x^2 + 70x}{x}$

$= Rs(\frac{1}{10}x^2 - 6x + 70)$

Average fixed Cost (AFC) $= \frac{FC}{x} = \frac{11}{x}$,

At the output level $x = 10$ tonnes

$AC = Rs(\frac{1}{10} \times 100 - 6 \times 10 + 70 + \frac{11}{10}) = Rs.\ 21.10$

$AVC = Rs.(\frac{1}{10} \times 100 - 6 \times 10 + 70) = Rs. 20$

$AFC = Rs. \frac{11}{10} = Rs. 1.10$.

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2. A firm produces x units of output per week at a total cost of Rs. $(\frac{1}{2}x^3 - x^2 + 5x + 3)$. Find the output levels at which the marginal cost and the average variable cost attain their respective minima.

Soln.:- Here Total Cost (TC) $= \frac{1}{2}x^3 - x^2 + 5x + 3$ (at the level of output x units)

variable cost $= \frac{1}{2}x^3 - x^2 + 5x$ (at the output level of x units)

Average variable cost (AVC) $= \frac{VC}{x} = \frac{1}{2}x^2 - x + 5$ ---(i)

Marginal Cost (MC) $= \frac{d}{dx}(TC) = \frac{3}{2}x^2 - 2x + 5$

$AVC = \frac{1}{2}x^2 - x + 5$

Differentiating both sides w.r.to x, we get

$\frac{d}{dx}(AVC) = \frac{2}{2}x - 1 = x - 1$ ---(ii)

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For maximum & minimum of AVC, $\frac{d}{dx}(AVC)=0$

or $x-1=0$

$\Rightarrow x=1$

Again differentiating both sides of (ii) w.r.to x, we have

$$\frac{d^2(AVC)}{dx^2} = 1 > 0 \text{ (+ve)}$$

So the AVC will be minimum ~~at~~ at the output level of 1 unit.

$$MC = \frac{3}{2}x^2 - 2x + 5 \quad \text{-----(iii)}$$

Differn. both sides w.r.to x we have

$$\frac{d}{dx}(MC) = \frac{3}{2}.2x - 2 \quad \text{-----(iv)}$$

For Maximum and minimum (MC), $\frac{d}{dx}MC = 0$

i,e $3x-2=0$

$\Rightarrow x = 2/3$ unit

Again diffn. both sides of (iv) w.r.to x, we have

$$\frac{d^2(MC)}{dx^2} = 3 > 0 \text{ (+ve)}$$

So, at the output level of 2/3 units the marginal cost will be minimum.

3. The demand function of a firm, is given by the relation $2p+3x=60$, where p is price per unit and x is the number of units demanded, Find the level of output which maximises the total revenue and also find maximum total revenue.

Soln.

$\because 2p+3x=60$

$\Rightarrow 2p = 60-3x$

(Demand funn.) $\Rightarrow p = 30 - \frac{3}{2}x \quad \text{---(i)}$

Total Revenue (R) = $px = 30x - \frac{3}{2}x^2 \quad \text{---(ii)}$

Diffn. both sides of (ii) w.r.to x, we get—

$$\frac{dR}{dx} = 30 - 3x \quad \text{---(iii)}$$

For maximum R, $\frac{dR}{dx}=0$ or

or $30-3x=0$

$\Rightarrow x=10$

$$\frac{d^2R}{dx^2} = -3 < 0 \ (-ve)$$

So, the total revenue R is maximum at $x=10$

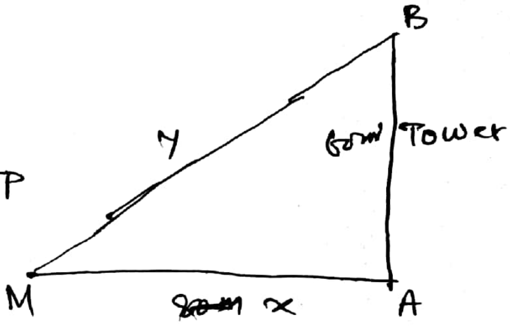
So, the required level of output is $x=10$.

and the maximum total revenue $= 30\cdot 10 - \frac{3}{2}\cdot 10^2$

$= 300 - 150$

$= \underline{150}$ Ans.

4. A man is walking at the rate of 5km/hr towords the foot of a tower 60 meter high. At what rate is he approaching the top of the tower when he is 80 meter from the foot of the tower?



Soln. :- Let BM = ym & AM = xm

A is the foot of the tower, B is the top

The man is at the point AM, approaching towords A.

From the right angled tringle AMB we have

$BM^2 = AB^2 + AM^2$ or $y^2 = x^2 + 60^2$ ...(i)

or $y^2 = 60^2 + 80^2$ $\therefore y = \sqrt{x^2+3600}$

$\therefore y = \sqrt{10000} = 100$ meter.

From (i) $\cancel{2y = 2x}$. $y^2 = x^2 + 60^2$

Diffn. both sides w.r.to t

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$\therefore y\frac{dy}{dt} = x\frac{dx}{dt}$$

$$\text{or } \frac{100}{1000}\frac{dy}{dt} = \frac{80}{1000}\times 5$$

$$\therefore \frac{dy}{dt} = 4$$

Alternative

$$y = (x^2+3600)^{1/2}$$

$$\therefore \frac{dy}{dt} = \frac{1}{2}(x^2+3600)^{-\frac{1}{2}}\frac{d}{dx}(x^2+3600).$$

$$= \frac{1}{2\sqrt{x^2-3600}}\cdot 2x \cdot \frac{dx}{dt}$$

$$= \frac{80}{\sqrt{6400+3600}}\times 5$$

$= 4$ km/h.

So the man will approaching towords the top of the tower at the rate of 4km/hour.

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